Session Ic: Hedging Guaranteed Benefits

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Discussion Overview

- The Greeks
- Risk Neutral vs. Real World
- Hedging Guaranteed Benefits
- Questions

The Greeks

A Look Behind The Curtains

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The Greeks

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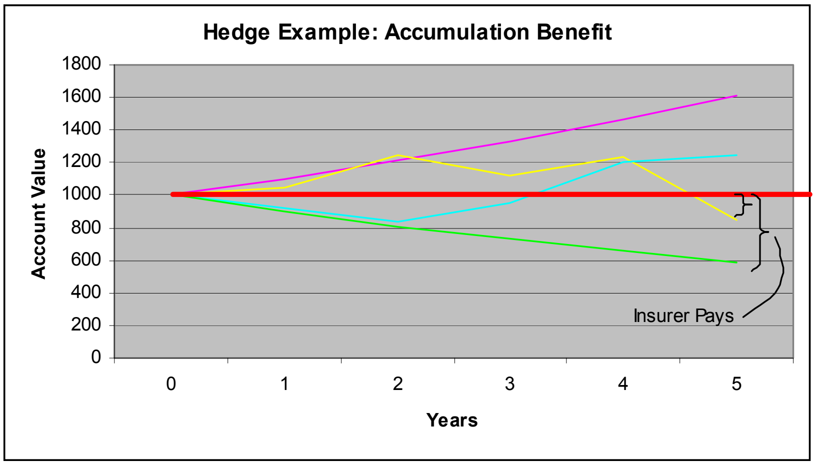
Why talk about Greeks?

- Insurance companies, consultants and analysts often describe hedge programs in terms of Greeks
  - "Three Greek Program"
  - "Best practice companies now hedge Delta, Vega and Rho"
- Goal is to understand the context and content of Greeks

Basics

- Greeks are used in the context of financial options
- Annuity writers are in the business of selling financial options
  - Need to value financial options and, if desired, hedge them
- What is hedging?
  - Reduce variability of the outcome of selling options
    - For example, turn a variable insurance payoff into a fixed cost that is known upfront

Hedge Example: GMAB



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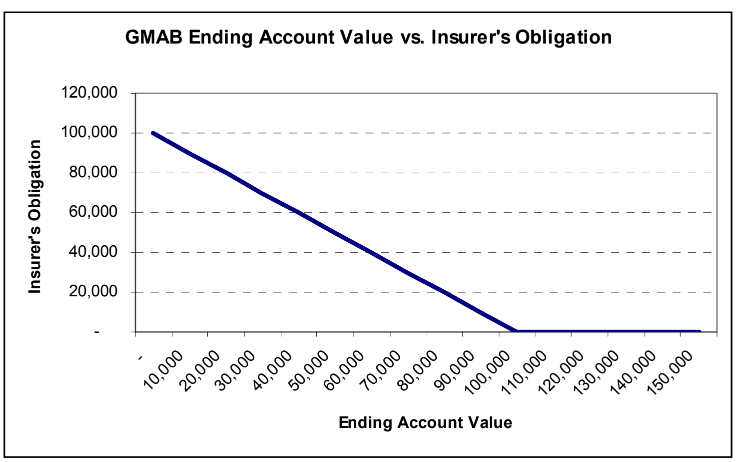
Hedge Example: GMAB



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So what is hedging?

- Insurer obligation is a financial option (a put option)
  - "Derived" from ending account value (a derivative)
- Hedging: Buy a derivative that gives the insurance company (IC) a cash flow equal to the IC's obligation
  - Take out all variability of results
    - Don't have to take out all variability…
  - Buy derivative upfront: known cost!

Generalize

- Hedge Theme: IC
  - Sells a financial option
  - Then buys back the financial option
- Approaches to buying back the option
  - Directly: go to Wall Street and buy it
  - Indirectly: make it yourself (= Dynamic Replication)

Greeks are about Manufacturing Options

Manufacturing Options

- = “Dynamic Replication”
- Idea:
  - Continually manufacture the change in the value of the option to be replicated
  - Key: cost of doing this is known upfront!
- Mechanics: Greek matching

The Greeks

- First, what is a Greek?
- Each Greek describes the *sensitivity* of an option value to changes in an underlying driver
- In mathematical terms, they are the *partial derivative* of the option value with respect to one of the underlying driver
- Price of the underlying: $\partial V/\partial S = \Delta$(Delta); $\partial^2 V/\partial S^2 = \Gamma$(Gamma)
  - Interest rate : $\partial V/\partial r = \rho$ (Rho)
  - Volatility: $\partial V/\partial \sigma$= *vega*

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Greek Matching

- **Greek Matching** is synthesizing the change in option value by selecting liquid financial instruments that have *matching sensitivity* to the underlying variables -- i.e., they have matching partial derivatives

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The Catch

- Greek matching only makes sense for the stochastic variables
  - In Black-Scholes framework, the only stochastic variable is **S**, the value of the underlying stock/equity
  - Interest rates (**r**) and volatility (**σ**) are parameters -- aka, assumptions
- To hedge variables other than **S**, these variables must first be modeled stochastically

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Delta and Gamma

- Remember: $\partial V/\partial S = \Delta$, $\partial^2 V/\partial S^2 = \Gamma$
- We will examine Delta and Gamma in terms of a put option
- Insurance Application: GMAB
  - IC promises to return initial premium at the end of the AB period
  - Implication: if the account value is below the initial premium at expiration, IC must make up the difference

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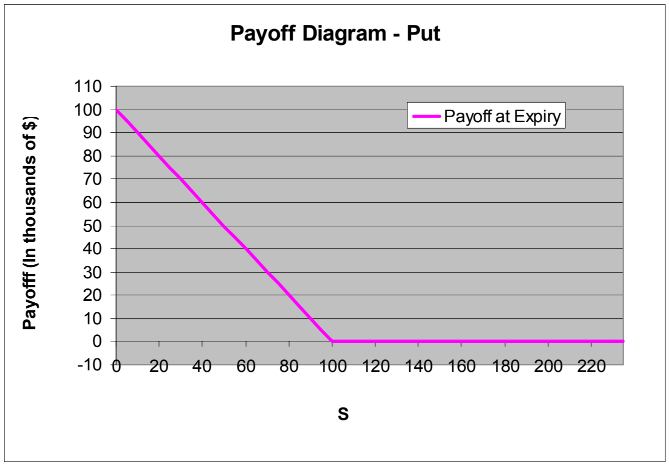
Put Payoff



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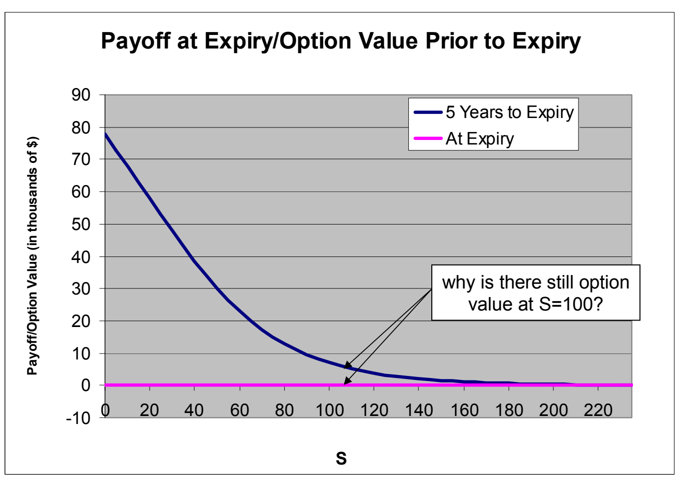
Put Payoff / Option value



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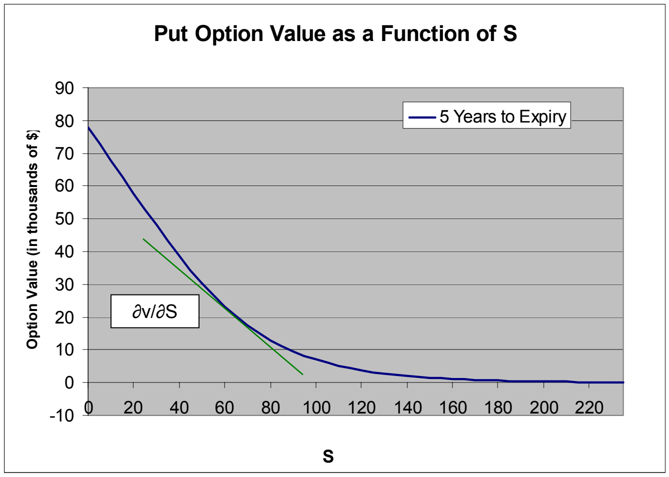
Delta – Geometric Intuition



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Dynamic Replication

- To dynamically replicate, we need to find financial instruments that replicate the sensitivity to S
  - That is, we need an instrument with matching Delta
- Example: Use Futures
  - A futures contract is an agreement for one party to sell a specific commodity to another party on a specific future date for a pre-set price
  - Many companies use futures to hedge exposure to S
  - Good for matching small changes in S
  - Catch: futures have 0 Gamma, which means that they are not a good fit for large changes in S

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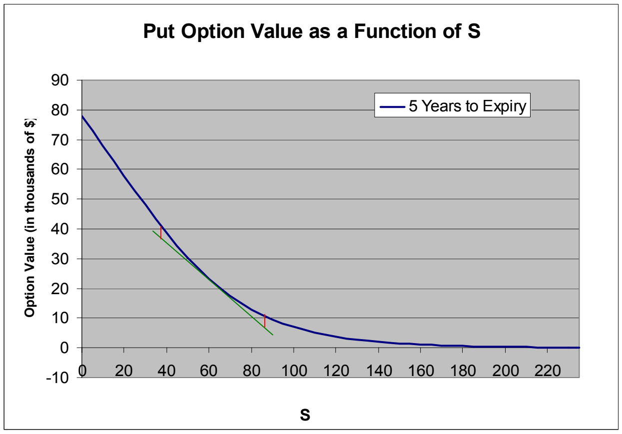
Why Gamma is Important

- Because this is dynamic replication, the errors are replicated every day. This means that Gamma errors don't need to be very big to really add up!



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Gamma

- To hedge against the effects of gamma, we must select instruments that give us both Delta and Gamma
- In this case, futures will not suffice (zero Gamma)
- The good news? We can also hedge with options, which have non-zero Gamma

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Risk Neutral vs. Real World

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Risk Neutral vs. Realistic World

- Risk Neutral World
  - Independent of risk preferences
  - Based on expected values of future payoffs
- Realistic World
  - It's not risks free
  - Investors are looking for more to compensate the extra risks they take on
  - Leads to risk-return tradeoff

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Risk Neutral Valuation

- In a 'risk neutral' world, all assets earn risk free rate on average
  - Compare this to Black-Scholes framework
- Adjust expected cash flows for risk and discount at risk-free rate
- Commonly used to determine the value of financial instruments where no closed form solution exists
- Catch: How many scenarios are needed before convergence?

Realistic Valuation

- As with any risky investment, an investor in an asset that is subject to market risk expects to earn a risk-free return plus a risk premium
- Discount expected cash flows at a risk-adjusted discount rate
- Catch: How to determine the expected cash flows and the risk premium?

Risk Neutral vs. Realistic Valuation

- Both approaches are mathematically equivalent
- For this reason, people use randomly generated scenarios from risk neutral economic scenario models to price complex derivative assets
- The risk neutral process generates scenarios in which all risk premium are zero
- When generating scenarios for reserve adequacy testing, or risk profile curves, people should not use risk neutral economic scenario generator

Categories of Scenario Generation Uses

- Risk Neutral Scenario
- Realistic/Real Scenario
- Realistic with Risk Neutral Valuation
- Realistic with Realistic Nesting
- Realistic with Realistic with Risk Neutral triple Nesting!

Risk Neutral Scenario Uses

- Commonly used in the following valuations
  - Duration and convexity
  - Valuation of embedded guarantees
  - Liability Greeks
  - Market value of assets
- Not for everything!
- Due to the large number of scenarios required, performing a risk neutral valuation can be run-time intensive

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Realistic/Real World Scenario Uses

- Scenario-based evaluation of portfolio strategies
- Generating scenarios for reserve adequacy testing
  - Real world has 'fat tails'
  - Distribution of 'tails' vital when assessing RBC requirements
  - Distribution of outcomes must be realistic
- Stress testing asset/liability strategies under adverse movements in interest rates
- PVDE risk profile curves
- Embedded Value at Risk
- Distribution of IRR

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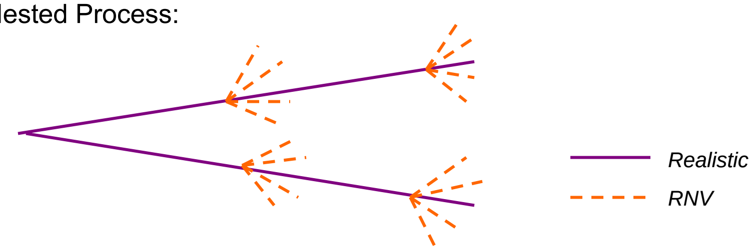
Pathwise Scenario Generation

- Ability to generate additional sets of stochastic scenarios at future time periods
- Typically use realistic scenarios to project experience
- Use risk neutral scenarios to determine hedge asset and liability values
- Nested Process:



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Realistic with Risk-Neutral for Valuation

- FAS 133
- Risk capital
- Dynamic rebalancing
  - Hedge positions
- Pathwise liability Greeks
  - Hedge strategy simulation

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Realistic with Realistic Nesting

- SOP 03-1
- AG 39
- Stochastic DAC

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VA Hedging – Liability Greeks Calculation

- Start with a validated deterministic model
- Determine the experience scenarios
- For point in time:
  - Generate the baseline risk neutral scenarios
  - Generate the "shocked" risk neutral scenarios
  - Run each scenario set and summarize results
  - Calculate the Greeks
- Pathwise Greeks
  - Employ nested processing to generate the risk neutral scenarios on-the-fly, summarize results and calculate Greeks.

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VA Hedging – Example

- Company A hedges its GMAB net delta exposure with S&P 500 futures:

GMIB	-10%	-1%	Index Base	1%	10%
Assets					
Futures MTM	480,153	48,015	-	(48,015)	(480,153)
Liability					
Revenue	2,617,138	2,764,861	2,780,395	2,795,828	2,932,818
Claims	1,615,364	1,286,279	1,254,596	1,223,747	979,145
Net Liability	1,001,774	1,478,582	1,525,799	1,572,082	1,953,673
Net Liability Change	(524,025)	(47,217)	-	46,283	427,874
MTM - Net Liability Change	(43,872)	799	-	(1,733)	(52,279)

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